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Energy spectra, intermittency and cross-velocity correlations in superfluid turbulence

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In collaboration with Anna Pomyalov

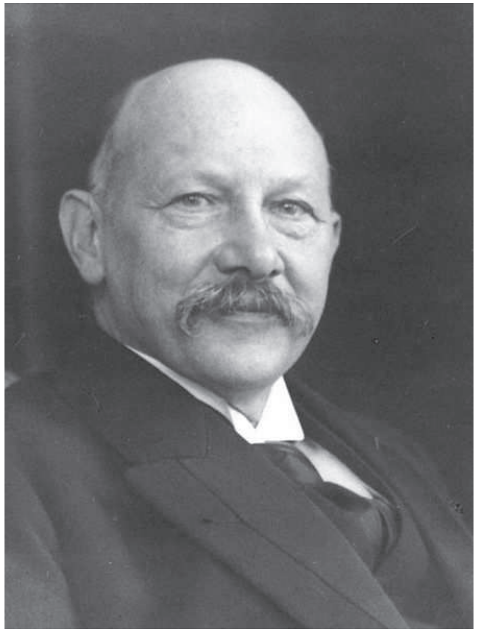
ABSTRACT

Turbulence in superfluid helium is unusual and presents a challenge to fluid dynamicists because it consists of two coupled, inter penetrating turbulent fluids: the first is inviscid with quantized vorticity, the second is viscous with continuous vorticity. Despite this double nature, the observed spectra of the superfluid turbulent velocity at sufficiently large length scales are similar to those of ordinary turbulence.

After brief historical overview I will present experimental, numerical and theoretical results which explain these similarities, and illustrate the limits of our present understanding of superfluid turbulence.

Superfluids: experiments and theory

Heike Kamerlingh-Onnes

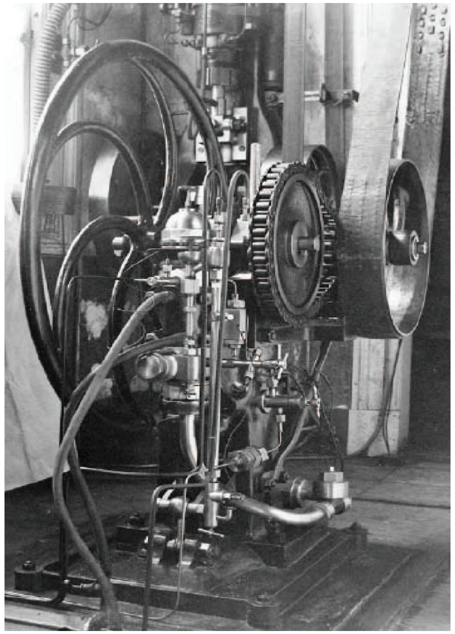


Nobel prize 1913

"for his investigations on the properties of matter at low temperatures which led, inter alia, to the production of liquid helium".

K-O discovered in 1911 superconductivity.

using this Compressor

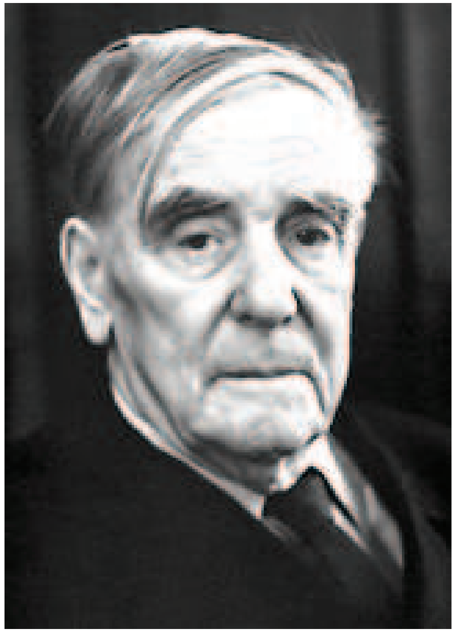


liquified He at $T = 4.2$ K in July 10, 1908.

K-O & coworkers in 1924 discovered density change at $T = 2.18$ K.

Keesom & Wolfke, 1928: this is a phase transition He I ⇔ He II.

Piotr Leonidovich Kapitza



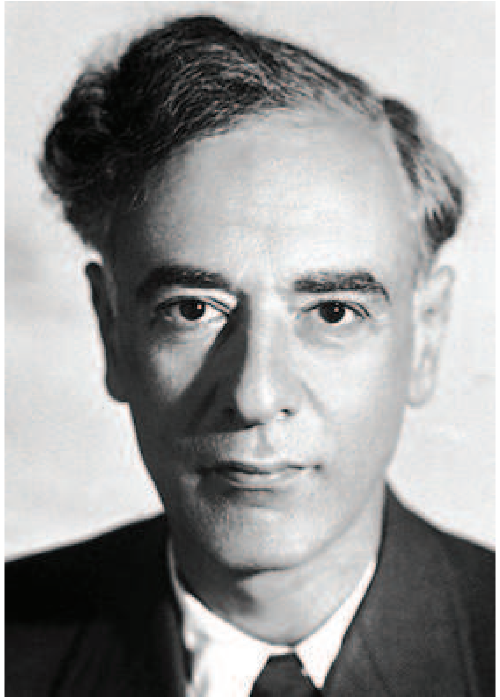
Nobel prize 1978

"for his basic inventions and discoveries in the area of low-temperature physics". P.L. Kapitza in Moscow discovered and named in 1937 superfluidity of ^{4}He

Jack Allen

and his student Donald Missener

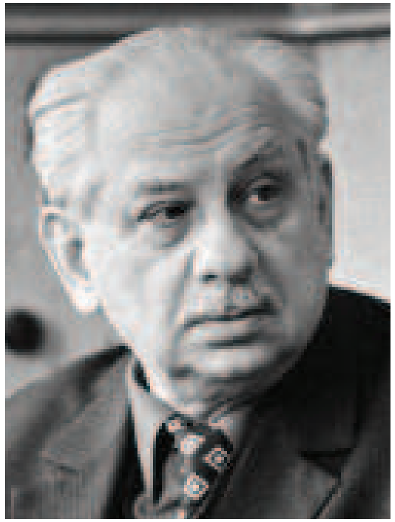
independently discovered superfluidity in PLK's Cambridge lab.



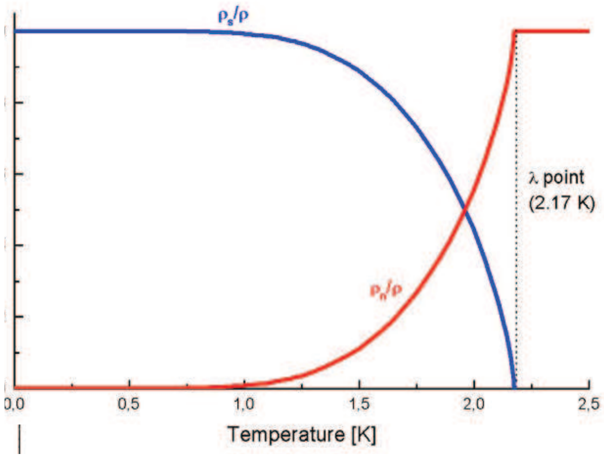
Lev Davidovich Landau
Nobel Prize, 1962
"for his pioneering theories for condensed matter, especially liquid helium".
In particular, he quantized in 1941 the Tisza-1940 two-fluid model and suggested Andronikashvilii's 1946 experiment on oscillating in He II discs.

Elepter Luarsabovich Andronikashvili

Laszlo Tizsa



Its period and damping measures densities of superfluid, $\boldsymbol{\rho}_{\rm s}$ and normal, $\boldsymbol{\rho}_{\rm s}$, components:



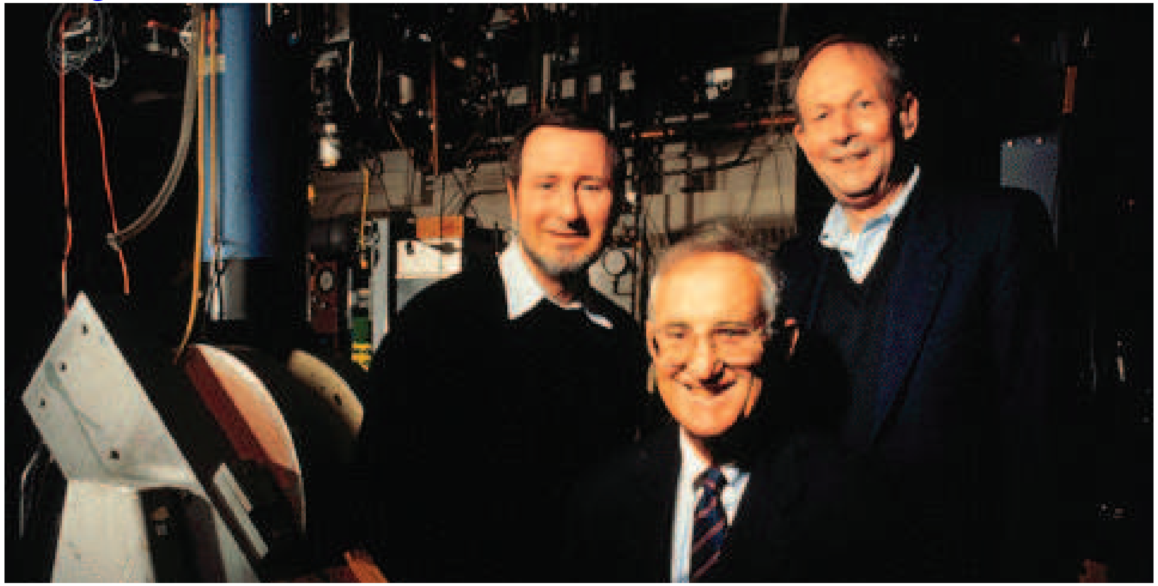
Landau-Tizsa two fluid model

for superfluid, $\boldsymbol{V}_n$, and normal $\boldsymbol{V}_s$ velocities:

$$\rho_s \frac{D\mathbf{v}_s}{Dt} = -\frac{\rho_s}{\rho}\nabla p + \rho_s S \nabla T - \mathbf{F}_{ns}$$

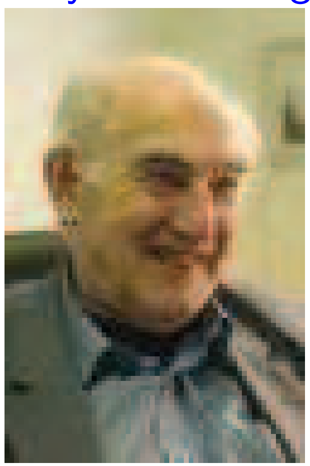
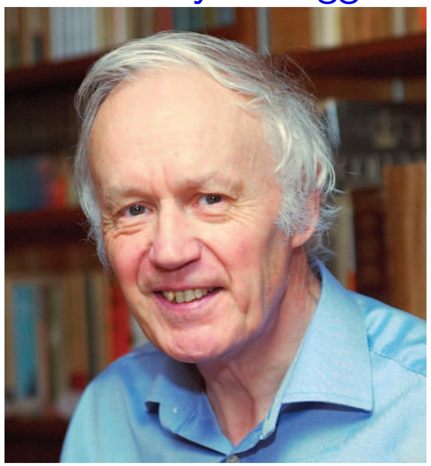
$$\rho_n \frac{D\mathbf{v}_n}{Dt} = -\frac{\rho_n}{\rho}\nabla p - \rho_s S \nabla T + \eta \nabla^2 \mathbf{v}_n + \mathbf{F}$$

predicts "second sound", critical velocity, etc.
Here: S – entropy, T – temperature and $\boldsymbol{F}_{ns} = A\rho_n\rho_s(\boldsymbol{V}_s - \boldsymbol{V}_n)^3$ is the mutual friction between superfluid and normal components



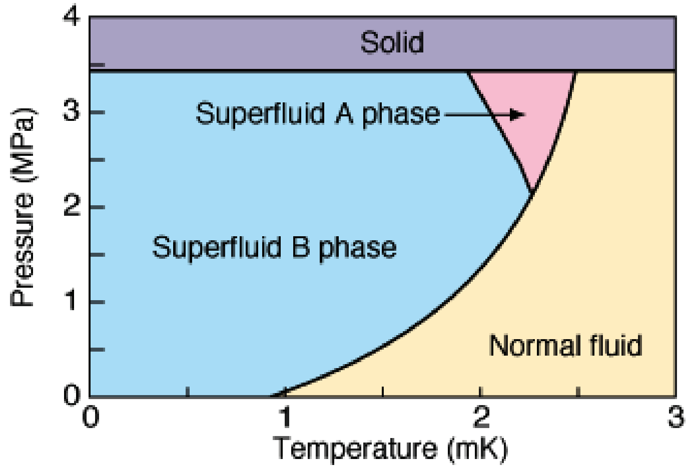
Nobel Prize 1996 "for their discovery of superfluidity in helium-3".

Alexei A. Abrikosov, Vitaly L. Ginzburg, & Anthony J. Leggett

Nobel Prize 2003 "for pioneering contributions to the theory of superconductors and superfluids"

produced (150 Kg since 1955) and liquified in LANL. Using Pomeranchuk's compressive cooling D.O, R.R&D.L discovered superfluidity of ^{3}He on April 20, 1972 at Cornell.



Knowing this before publication, J. Leggett on Sept. 5, 1972 submited to PRL explanation of their observations as Bardeen-Cooper-Schrieffer condensation of Couper pairs of ^{3}He atoms in the triplet state with the tensorial ordering parameter. The B-state has an isotropic gap.

Quantum mechanical description of He II

Macroscopic wave function $\Psi = \sqrt{\rho_s}\,\exp\{i\varphi(r,t)\}$

$$\hat{p} = i\hbar\nabla \longrightarrow \mathbf{v}_s = \frac{\hbar}{m_4}\nabla\varphi \longrightarrow curl\,\mathbf{v}_s = 0$$

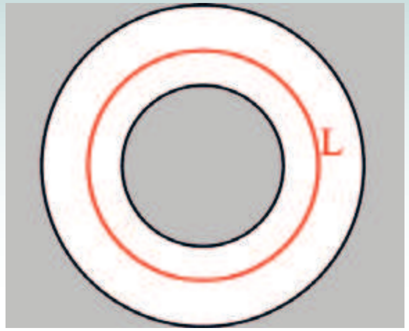
Circulation –singly connected region

$$\Gamma = \oint_L \mathbf{v}_s d\ell = 0$$

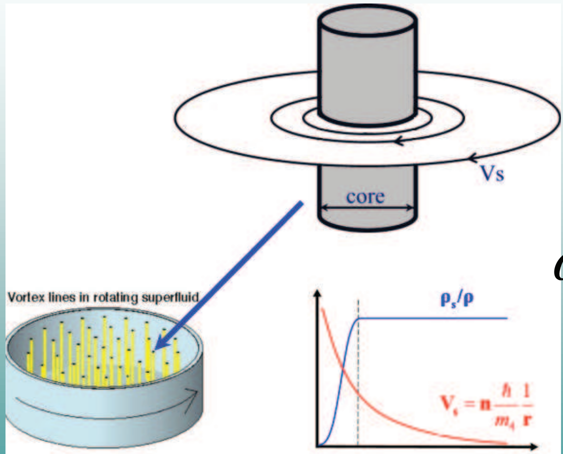
Circulation- multiply connected region

$$\Gamma = \oint_L \mathbf{v}_s d\ell = n\frac{h}{m_4} = n\kappa$$

$$\kappa \cong 10^{-7}\,m^2/s$$



Quantized vortices in He II

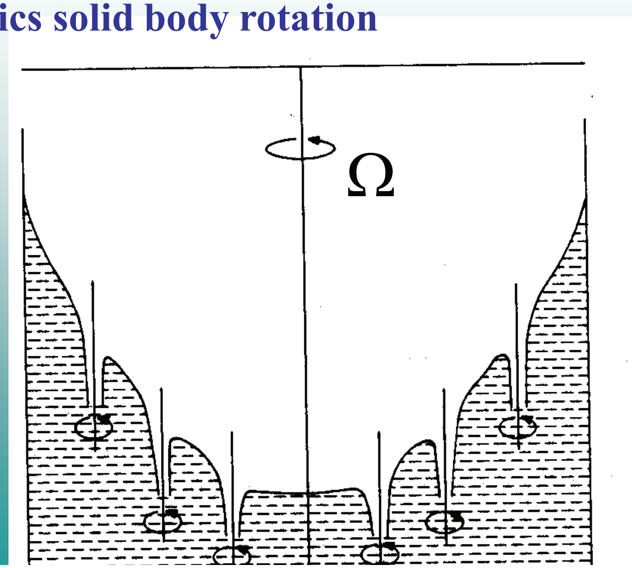


vorticity

$$\omega_N = 2\Omega \cong \langle \omega_S \rangle \cong \kappa L$$

Rotating bucket of He II

-thanks to the existence of rectilinear vortex lines He II mimics solid body rotation



Turbulence in a superfluid was predicted first by Richard Feynman in **1955** and found experimentally (in counterflow ^{4}He) by Henry Hall and Joe Vinen in **1956**.

Consider

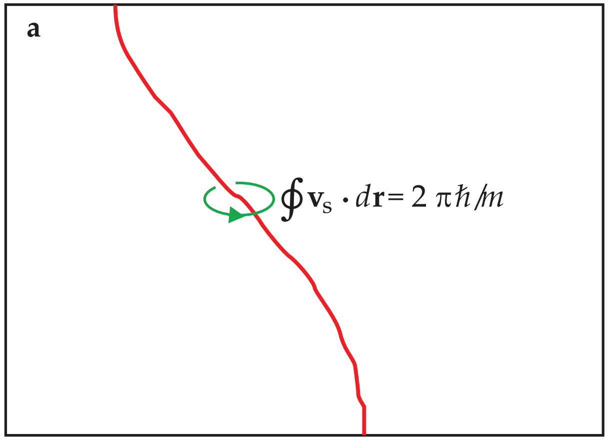
1.3.1 Normal fluid *vs.* superfluid at $T \to 0$ limit:

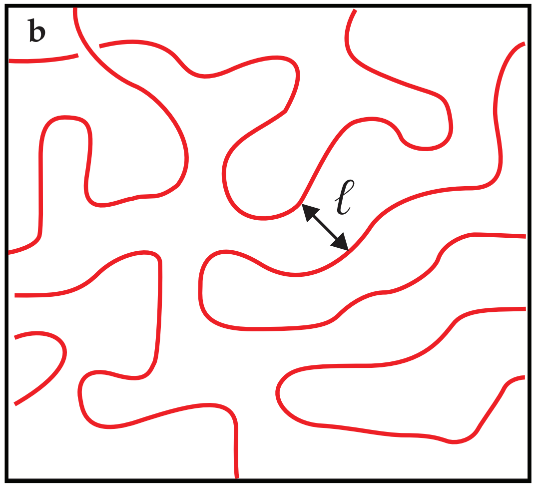
– Normal fluid kinematic viscosity $\nu \neq 0$ *vs.* $\nu \equiv 0$ in superfluids;

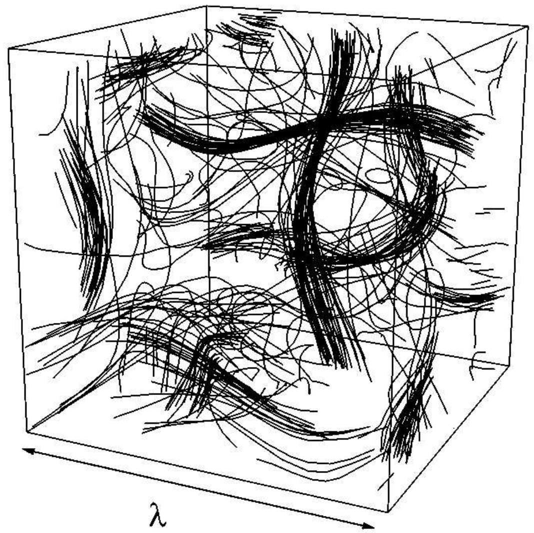
– Two scales in normal fluids: Outer scale L and dissipative micro-scale $\eta \ll L$;

– Two additional scales in superfluids due to quantization of vortex lines:

$\Downarrow$ vortex core diameter a_0 $\Downarrow$ $\quad$ $\Downarrow$ mean inter-vortex distance ℓ $\Downarrow$ $\quad$ $\Downarrow$ Outer scale D $\Downarrow$

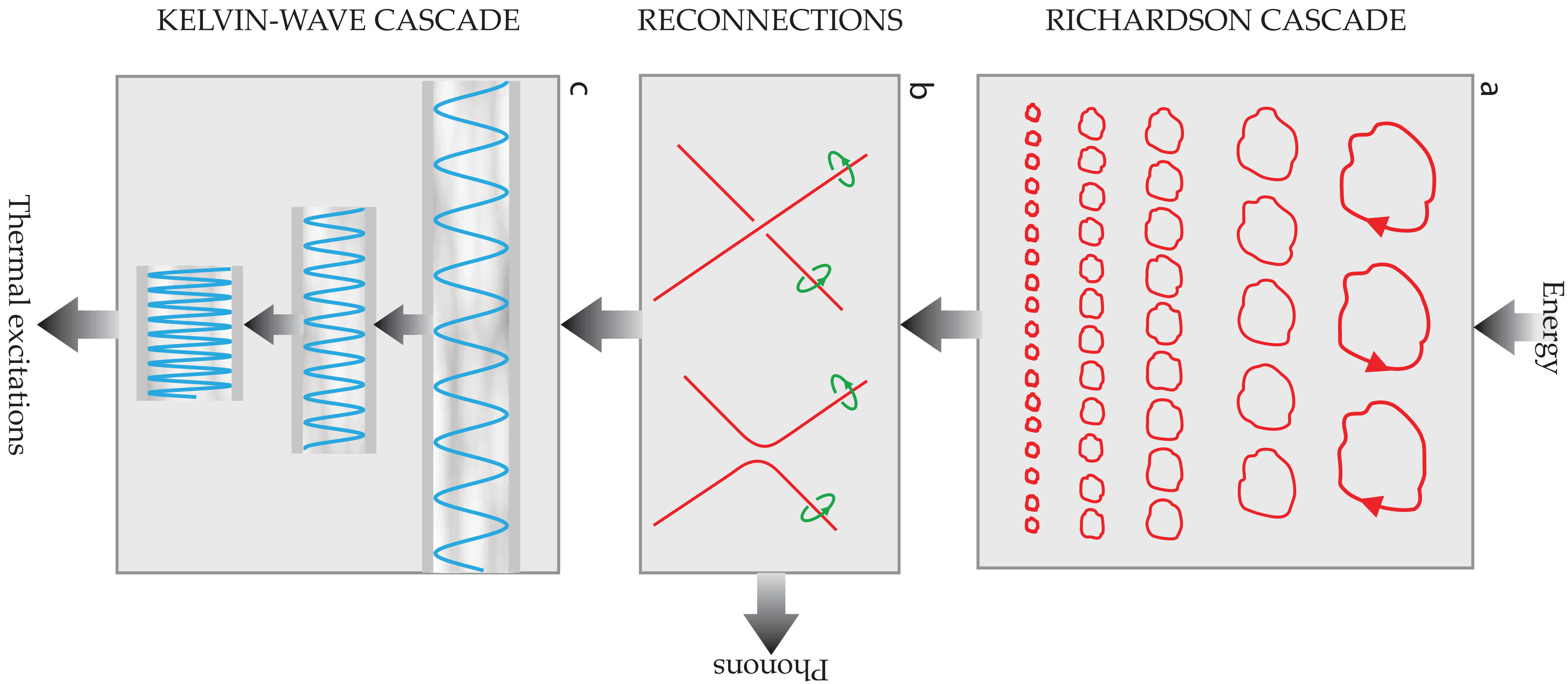






In ^{4}He $a_0 \simeq 1$ Å, in ^{3}He $a_0 \simeq 800$ Å. Experimentally, in both ^{4}He and ^{3}He, $\Lambda \equiv \ln\left(\frac{\ell}{a_0}\right) \simeq 12 \div 15$

Sketch of the quantum-turbulence cascades[1]



Very small scales: $L \ll \ell$.

Intervortex scales: $L \sim \ell$

Large scales: $L \gg \ell$

Two-fluid hydrodynamic (HD) equations for medium and low temperatures, $T \gtrsim 0.8\,\mathrm{K}$

In the HD region, $R \gg \ell$, one can neglect the quantization of vortex lines and make use coarse-grained, two fluid Hall-Vinen-Bekarevich-Khalatnikov (HVBK) equation for velocities the superfluid and normal components $\boldsymbol{u}_{\rm s}$ and $\boldsymbol{u}_{\rm n}$, with densities $\rho_{\rm s}$ and $\rho_{\rm n}$ and pressures $p_{\rm s}$ and $p_{\rm n}$

$$\rho_{\rm s}\Big[\frac{\partial\,\boldsymbol{u}_{\rm s}}{\partial t}+(\boldsymbol{u}_{\rm s}\boldsymbol{\nabla})\boldsymbol{u}_{\rm s}\Big]-\boldsymbol{\nabla}p_{\rm s} = -\boldsymbol{F}_{\rm ns}\,, \quad p_{\rm s}=\frac{\rho_{\rm s}}{\rho}[p-\rho_{\rm n}|\boldsymbol{u}_{\rm s}-\boldsymbol{u}_{\rm n}|^2]\,, \tag{1a}$$

$$\rho_{\rm n}\Big[\frac{\partial\,\boldsymbol{u}_{\rm n}}{\partial t}+(\boldsymbol{u}_{\rm n}\boldsymbol{\nabla})\boldsymbol{u}_{\rm n}\Big]-\boldsymbol{\nabla}p_{\rm n} = \rho_{\rm n}\nu\Delta\boldsymbol{u}_{\rm n}+\boldsymbol{F}_{\rm ns}\,, \quad p_{\rm n}=\frac{\rho_{\rm n}}{\rho}[p+\rho_{\rm s}|\boldsymbol{u}_{\rm s}-\boldsymbol{u}_{\rm n}|^2]\,, \tag{1b}$$

coupled by the the mutual friction between superfluid and normal components of the liquid mediated by quantized vortices which transfer momenta from the superfluid to the normal subsystem and vice versa:

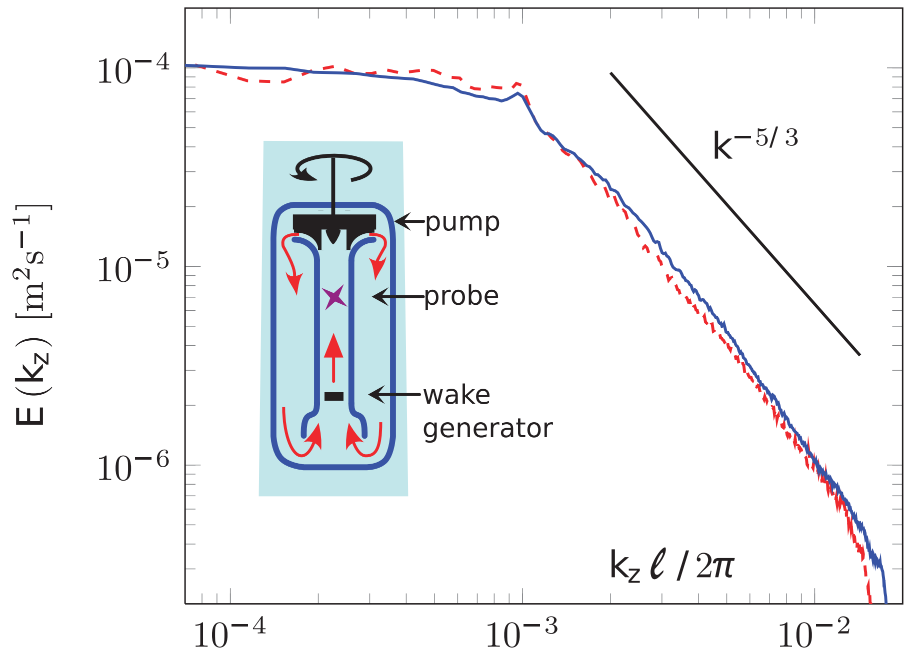
$$\boldsymbol{F}_{\rm ns}\approx \alpha\,\rho_{\rm s}\kappa\mathcal{L}(\boldsymbol{u}_{\rm s}-\boldsymbol{u}_{\rm n})\,, \quad \mathcal{L} \text{ is the vortex line density .} \tag{1c}$$

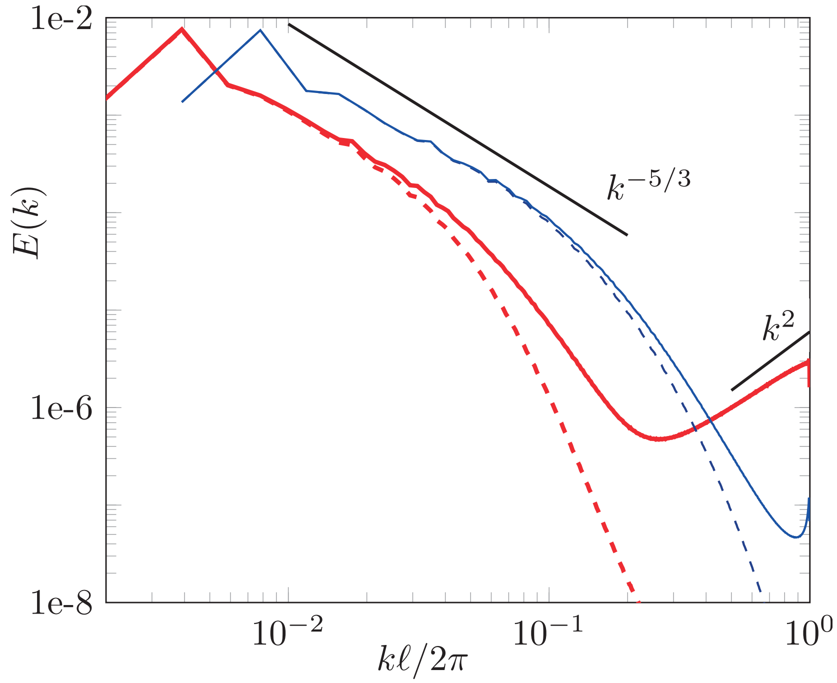
Eqs (1) are very similar to the Navier-Stokes equation. Therefore in a theory of large-scale superfluid turbulence we can use numerous tools, developed in the theory of classical HD turbulence, in particular, direct numerical simulations (DNS) and the differential closure for the energy flux

$$\boldsymbol{\varepsilon(k)=-\frac{1}{8}\sqrt{k^{11}\,E(k)}\,\frac{d}{dk}\Big[\frac{E(k)}{k^2}\Big]} \qquad \Rightarrow \qquad \boldsymbol{E(k)=k^2\Big[\frac{24\,\varepsilon}{11k^{11/2}}+\Big(\frac{T}{\pi\rho}\Big)^{3/2}\Big]^{2/3}}\,. \tag{2}$$

This solution with the constant energy flux $\varepsilon(k)=\varepsilon$ gives KO-41 spectrum $\propto k^{-5/3}$ for small k end thermodynamic equilibrium spectrum $T/\pi\rho$ at large k.

0.3 Kolmogorov spectra in ^{4}He turbulence





Energy spectrum measured in the TOUPIE wind tunnel (Inset) below the superfluid transition (solid blue line, $T = 1.56$ K and aboveT_λ (dashed red line) [1]

Right: Numerical energy spectra of the superfluid (solid lines) and normal (dashed lines) component from two-fluid Eqs. at $T = 1.15$ K (red) and $T = 2.157$ K (blue) with truncation of phase space beyond the intervortex scale [2]

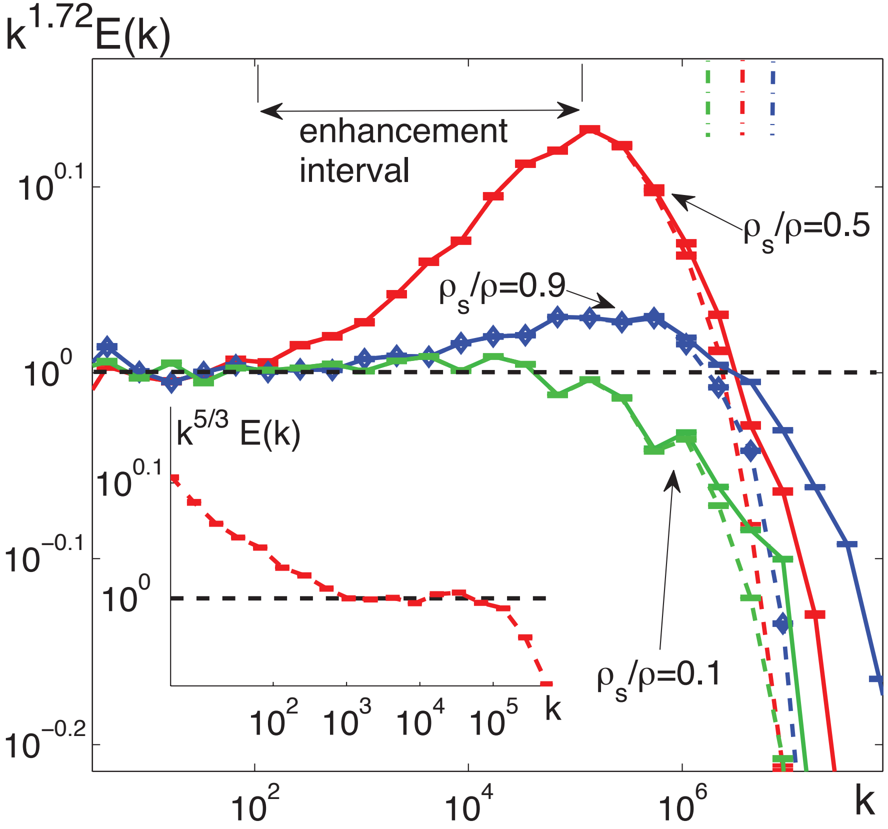
[1] Salort J, Chabaud B, Lvque E, Roche P-E, Energy cascade and the four-fifths law in superfluid turbulence. Europhys Lett. 97, 34006 (2012)

[2] C. F. Barenghi, V. S. Lvov, and P.-E. Roche, Experimental, numerical, and analytical velocity spectra in turbulent quantum fluid, Proc Natl Acad Sci USA., 111 46834690 (2014)

0.4 Intermittency enhancement in ^{4}He turbulence

L. Boue, V.S. L'vov, A. Pomyalov, I. Procaccia, PRL, 110, 014502 (2013)

Our shell model simulations with eight decades of k-space allowed detailed comparison of classical and superfluid turbulent statistics in the wide temperature range. A difference between classical and superfluid intermittent behavior in a wide (up to three decades) interval of scales was found in the range $0.8T_\lambda < T < 0.9T_\lambda$, where ($\rho_s \approx \rho_n$)

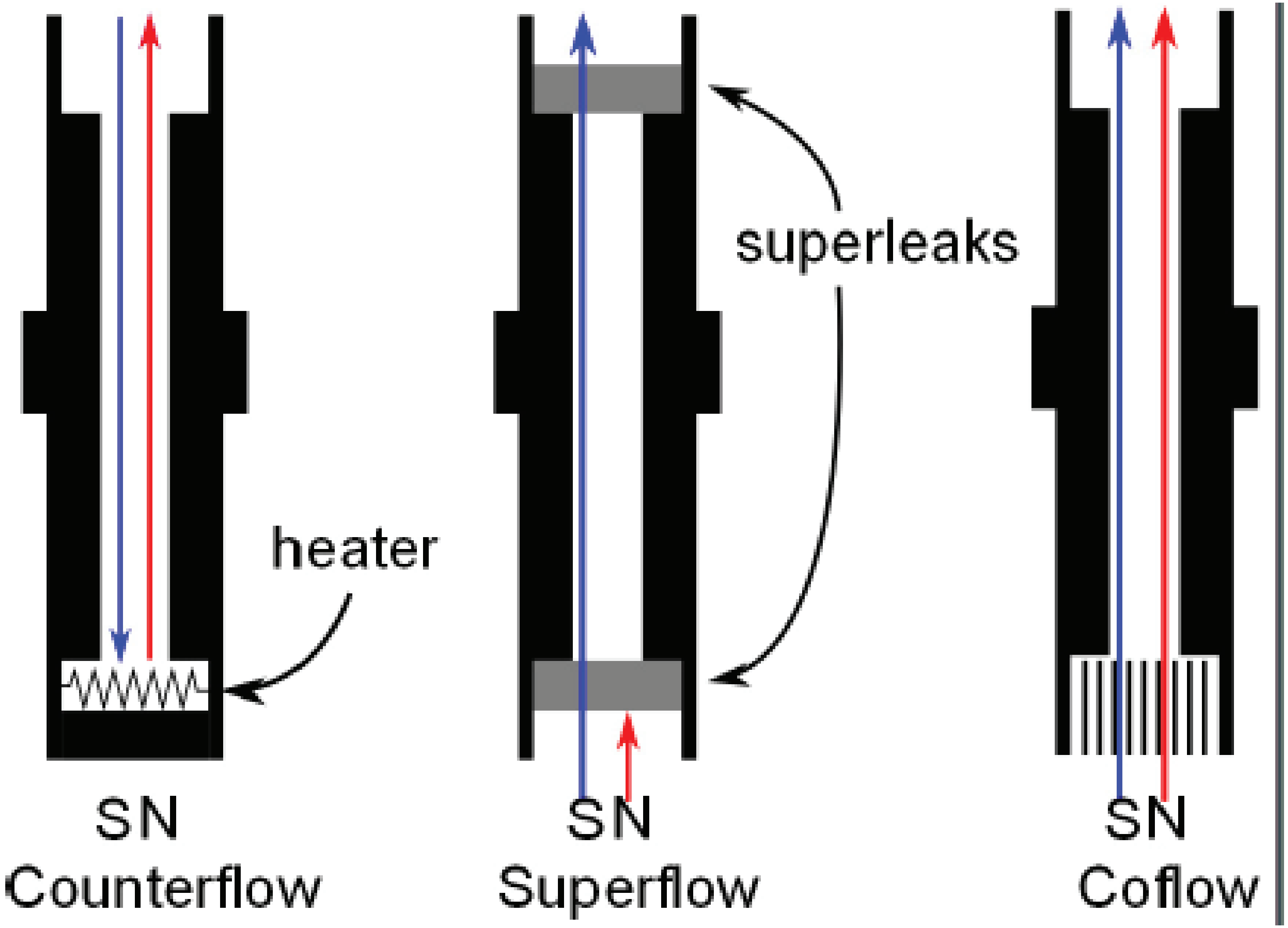


Superfluid (solid lines) and normal fluid (dash lines) compensated energy spectra $k^{1.72}E(k)$; the compensation factor is the classical energy spectrum with intermittency correction.

Inset: $k^{5/3}E(k)$ for $T = 0.9\,T_\lambda$. Shell model simulation of Eqs. (1) at $T/T_\lambda = 0.99$ K(green), 0.9(red) and 0.85(blue), corresponding to $\rho_s/\rho = 0.1\,,\ 0.5\,,$ and 0.9 respectively. The vertical dash lines indicate $k_\ell \equiv 1/\ell$.

DNS of HVBK two-fluid equations is very important for studies of intermittency effects in superfluid turbulence

Ladik Skrbek (Prague) channels for the study of counter-flow, super- and co-flow.[3]

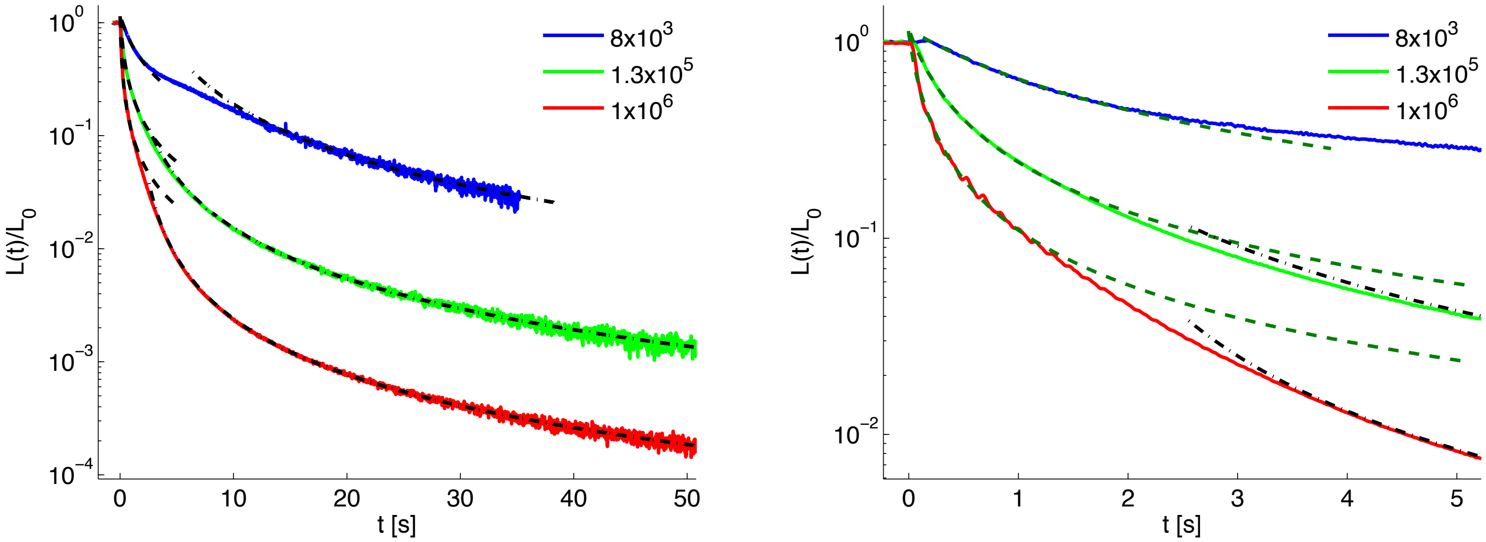


S and N stand for super-fluid and normal components. Counter-flow is produced thermally by a heater. Super-flow and co-flow are driven mechanically by a bellows.

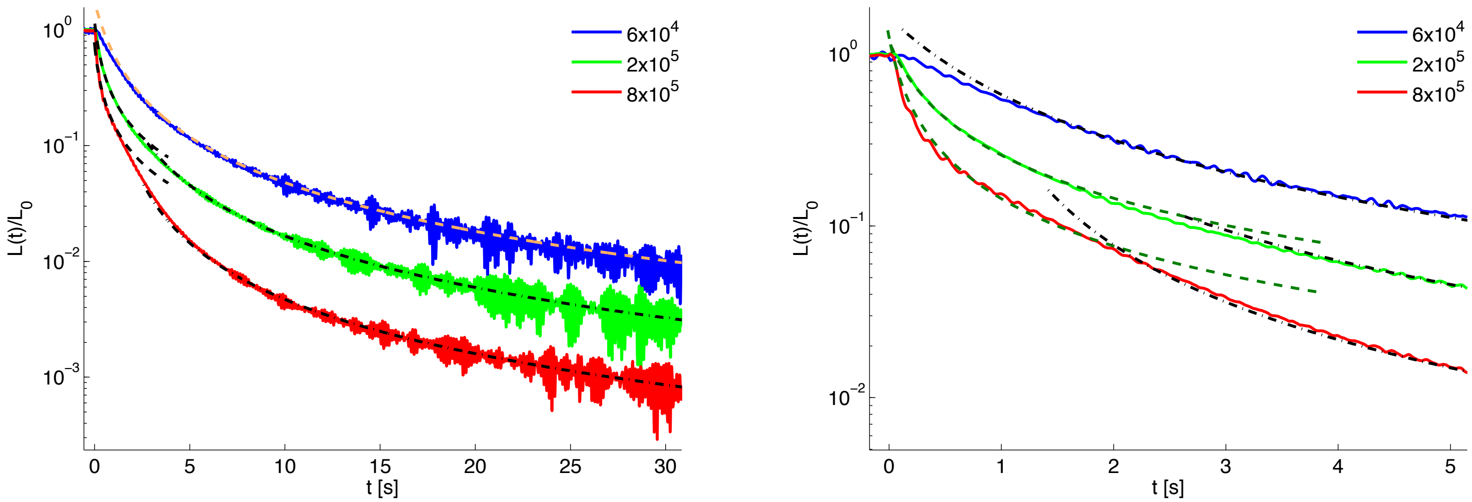
[3] **S. Babuin, V.S. L'vov, A. Pomyalov, L. Skrbek and E. Vargag[†], Coexistence and interplay of quantum and classical turbulence of superfluid ^{4}He, arXiv:1509.03765 (Sept. 2015)**

Prague data for the VLD decay $\mathcal{L}(t)/\mathcal{L}(0)$ in the 7 mm co-flow channel

$T = 1.35$ K



$T = 1.45$ K.



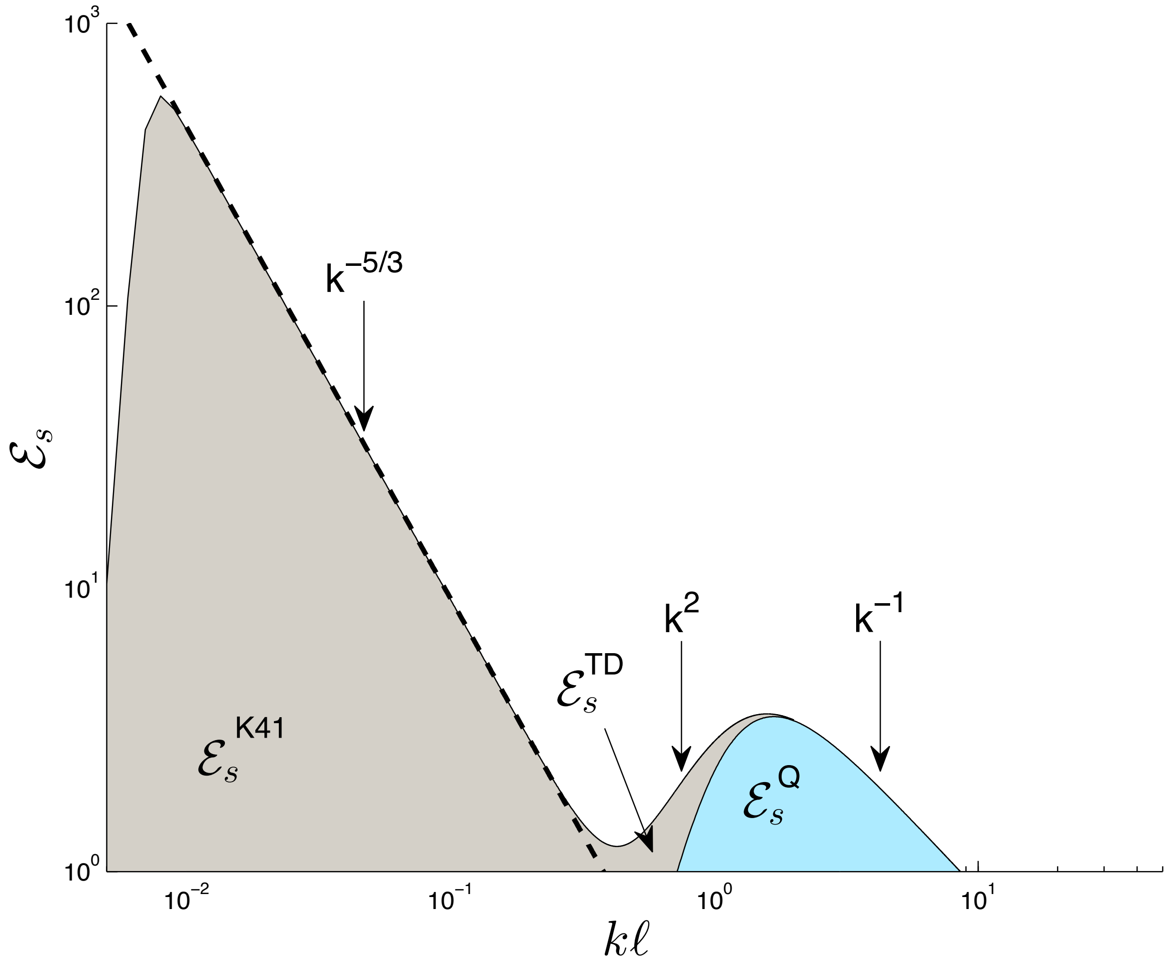
Quantum t^{-1}-fits – green dashed lines, classical $t^{-3/2}$-fits – black dash-dotted lines.

Coexistence of the classical (grey) and quantum (cyan) turbulence in co-flow

Classical energy spectrum consists of cascade part $\mathcal{E}_{\rm s}^{\rm K41}(k) \propto k^{-5/3}$

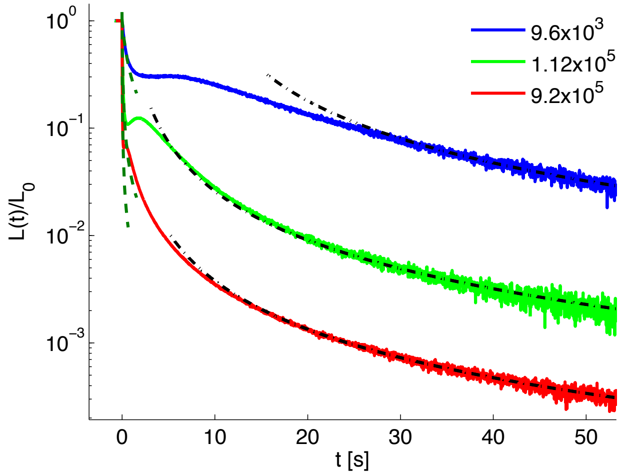
and thermodynamic equilibrium part $\mathcal{E}_{\rm s}^{\rm TD}(k) \propto k^{2}$

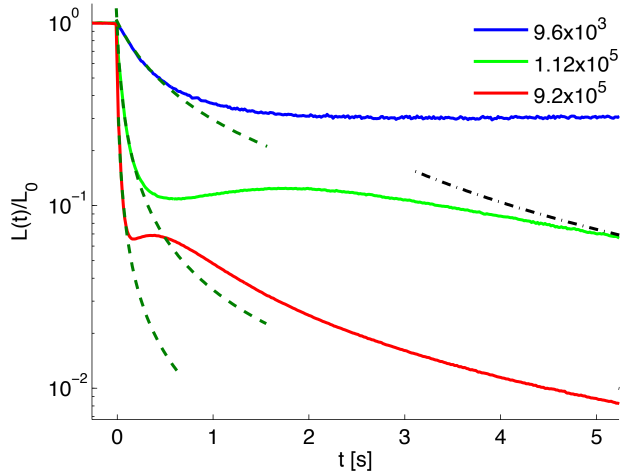
Quantum energy spectrum of random tangle has $1/k$ large k-asymptotics



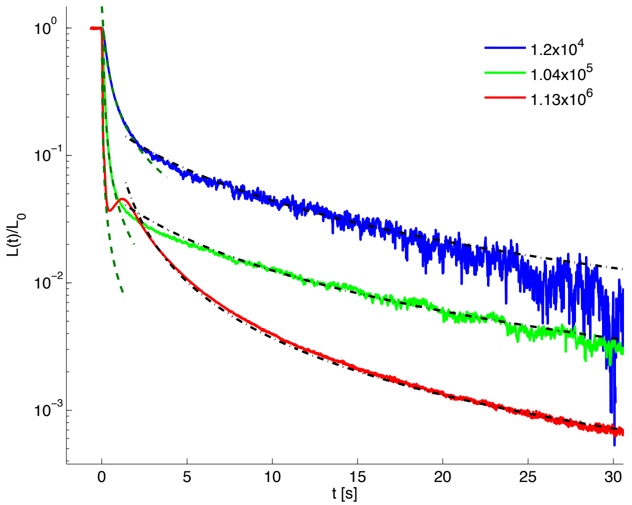
Prague data for the VLD decay $\mathcal{L}(t)/\mathcal{L}(0)$ in $10\,$mm, channel at $\boldsymbol{T = 1.45}\,$K

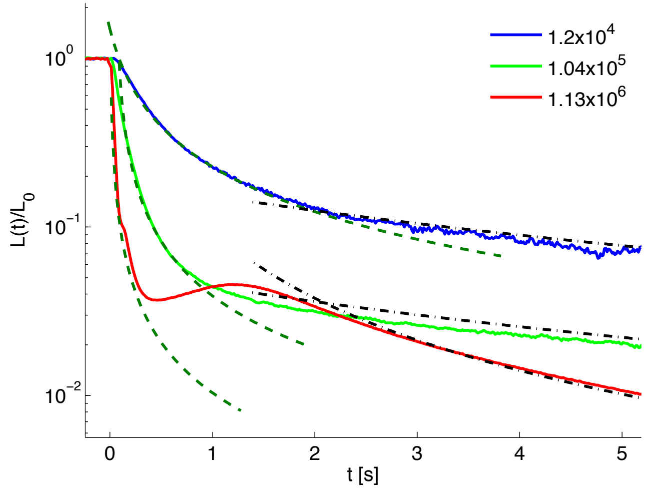
Counter-flow: Quantum t^{-1}-fits – green dashed lines, classical $t^{-3/2}$-fits – black dash-dotted lines.





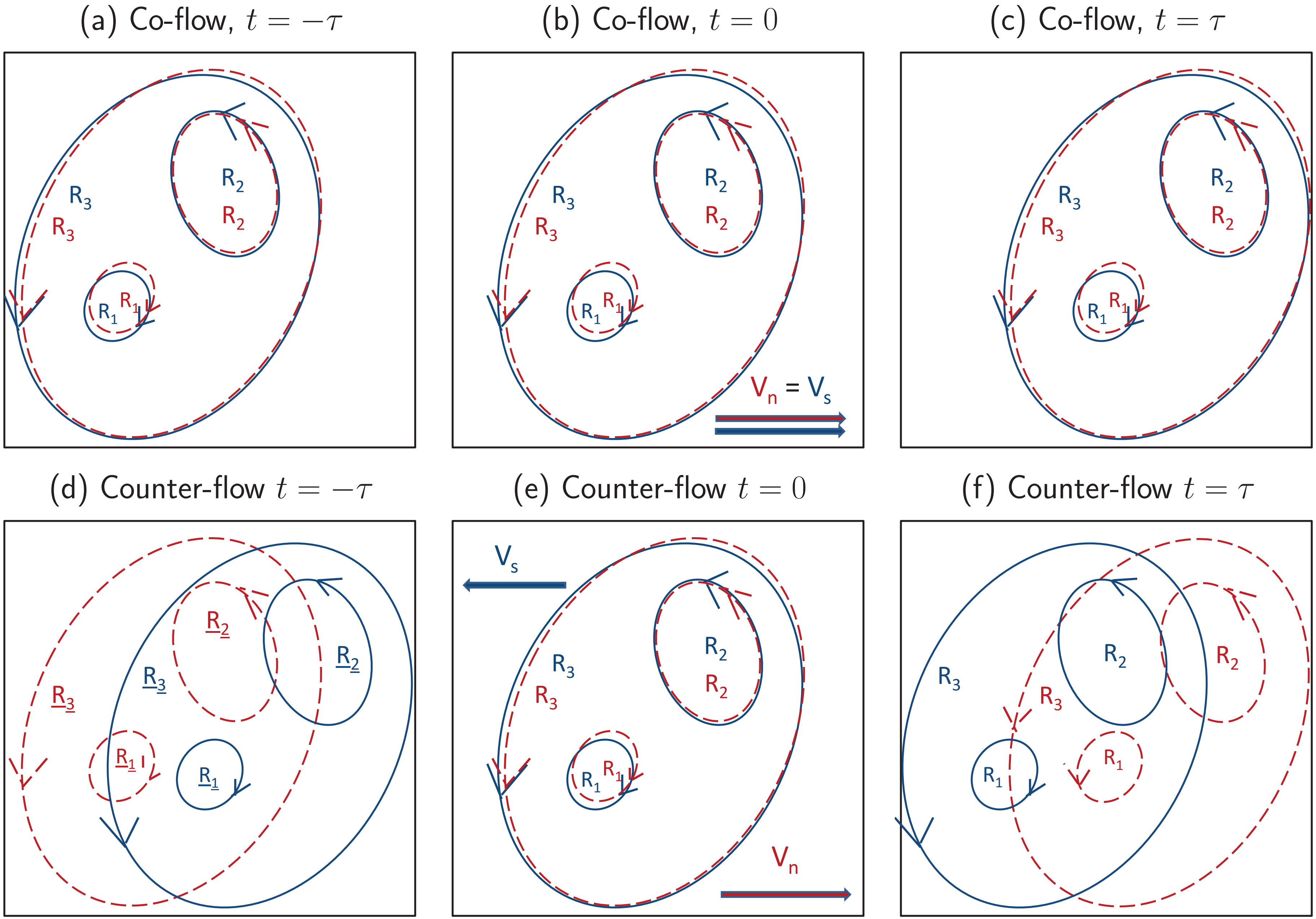
Superflow demonstrates behavior ⇓⇓ very similar to that of counter-flow ⇑⇑





A way to understand "bump" is to assume delay in the classical-energy supply of quantum tangle

Normal- (blue) & super-fluid (red) eddies swept by the mean normal- & super-fluid velocities $U_{\rm n}$ & $U_{\rm s}$



Time $\tau \simeq R_2/U_{\rm ns}$ is of the order of overlapping time of the middle-scale R_2-eddies.

Stationary energy spectra of counter- and pure super-flow turbulence

as a consequence of mutual-friction suppression due to

Counterflow decoupling of the normal- and super-fluid velocities

– Interaction (overlapping) time of scale R-eddies: $\boldsymbol{\tau_{\rm int}} = R/U_{\rm ns}$ ($\boldsymbol{U}_{\rm ns}$ – Counter-flow velocity)

– Mutual friction coupling time:

kind of the HVBK eqs:

$$\frac{\partial \boldsymbol{u}_{\rm s}}{\partial t} + \dots \simeq \alpha\kappa\mathcal{L}\left[\boldsymbol{u}_{\rm n}(\boldsymbol{r},t) - \boldsymbol{u}_{\rm s}(\boldsymbol{r},t)\right], \quad (3)$$

$$\frac{\partial \boldsymbol{u}_{\rm n}}{\partial t} + \dots \simeq -\alpha\frac{\rho_{\rm s}}{\rho_{\rm n}}\kappa\mathcal{L}\left[\boldsymbol{u}_{\rm n}(\boldsymbol{r},t) - \boldsymbol{u}_{\rm s}(\boldsymbol{r},t)\right], \quad (4)$$

$$\boldsymbol{u}_{\rm ns} \equiv \boldsymbol{u}_{\rm n} - \boldsymbol{u}_{\rm n}\,, \quad \boldsymbol{\Omega_{\rm mf}} \equiv \frac{\alpha\rho}{\rho_{\rm s}}\kappa\mathcal{L}\,, \qquad \frac{\partial \boldsymbol{u}_{\rm ns}}{\partial t} + \dots \simeq -\Omega_{\rm mf}\left[\boldsymbol{u}_{\rm ns}(\boldsymbol{r},t)\right]. \quad (5)$$

– Counterflow decoupling parameter $\boldsymbol{\zeta(R) = 1/\tau_{\rm int}\Omega_{\rm mf}} \;\Rightarrow \boldsymbol{\zeta(k) = \dfrac{kU_{\rm ns}}{\Omega_{\rm mk}}}$

Analytical theory of the coupling-decoupling processes, developed in Ref.[2] results in the equation for the dimensionless decoupling function $\mathcal{D}(k)$, which depends on k via decoupling parameter $\zeta(k)$:

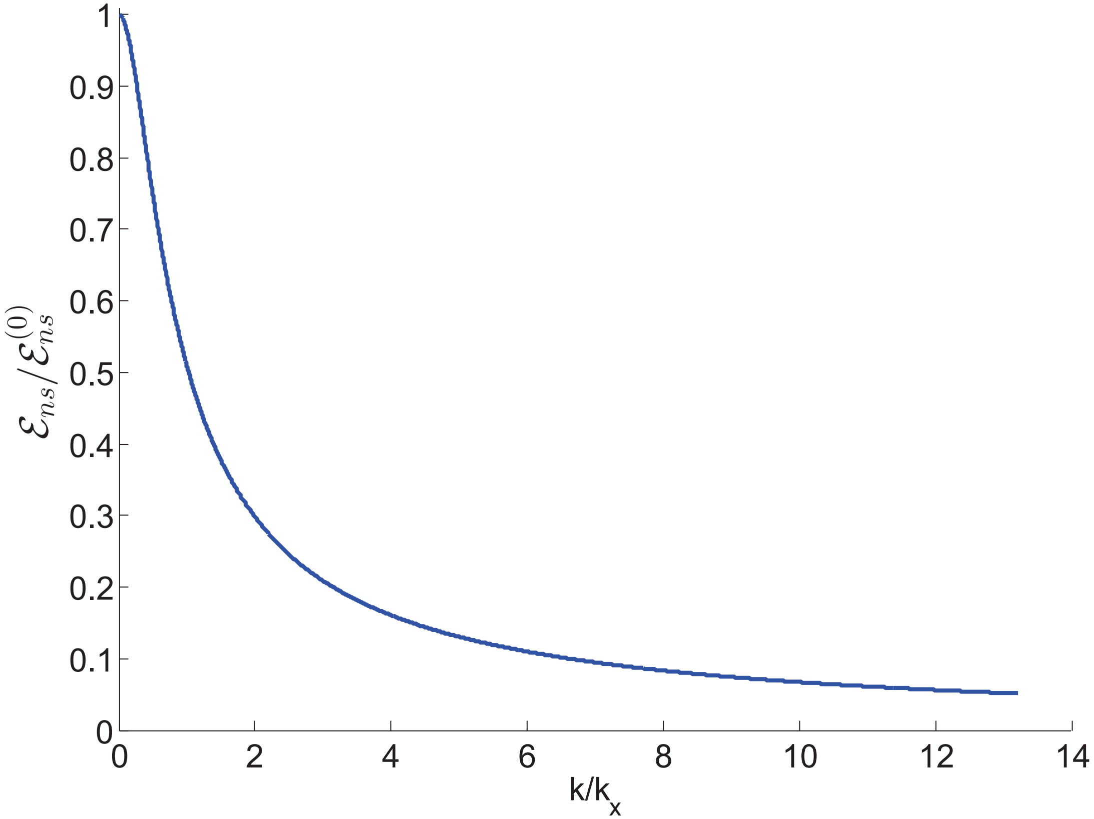
$$\boldsymbol{\mathcal{D}(k) = D[\zeta(k)] \equiv \frac{E_{\rm ns}(k, U_{\rm ns})}{E_{\rm ns}(k, 0)} = \frac{\arctan[\zeta(k)]}{\zeta(k)}}\,. \quad (6)$$

Here $E_{\rm ns}(k, U_{\rm ns}) = \langle \boldsymbol{u}_{\rm s}(\boldsymbol{k}) \cdot \boldsymbol{u}_{\rm n}(\boldsymbol{k})\rangle$ is cross-correlation function of the normal- and superfluid velocities in Fourier $\boldsymbol{k}$-representation.

$$D(\zeta) = 1 - \frac{\zeta^2}{3}, \text{ for } \zeta \ll 1\,, \qquad D(\zeta_\times) = \frac{1}{2}, \text{ for } \zeta_\times \approx 2\,, \quad D(\zeta) = \frac{\pi}{2\,\zeta}, \text{ for } \zeta \gg 1\,. \quad (7)$$

k-dependence of the decoupling function $\boldsymbol{D(k)}$

4



$\zeta_{\rm max} = \zeta(k_{\rm max})$ at the highest value of k, $k_{\rm max} \simeq \pi/\ell$.
With $\ell \simeq 1/\sqrt{\mathcal{L}} \simeq 1/(\gamma_{\mathcal{L}} U_{\rm ns})$ this gives a simple $U_{\rm ns}$-independent estimate of $\zeta_{\rm max}$:

$$\boldsymbol{\zeta_{\rm max} \simeq \frac{\pi}{\alpha_{\rm ns}\, \kappa\, \gamma_{\mathcal{L}}} \sim 40\,,}$$

Estimate:

$$\boldsymbol{D_{\rm min} = D(\zeta_{\rm max}) \simeq 0.04\,,}$$
$$\boldsymbol{\frac{k_{\rm max}}{k_\times} \simeq 20\,,}$$

for $T = 1.45\,$K.

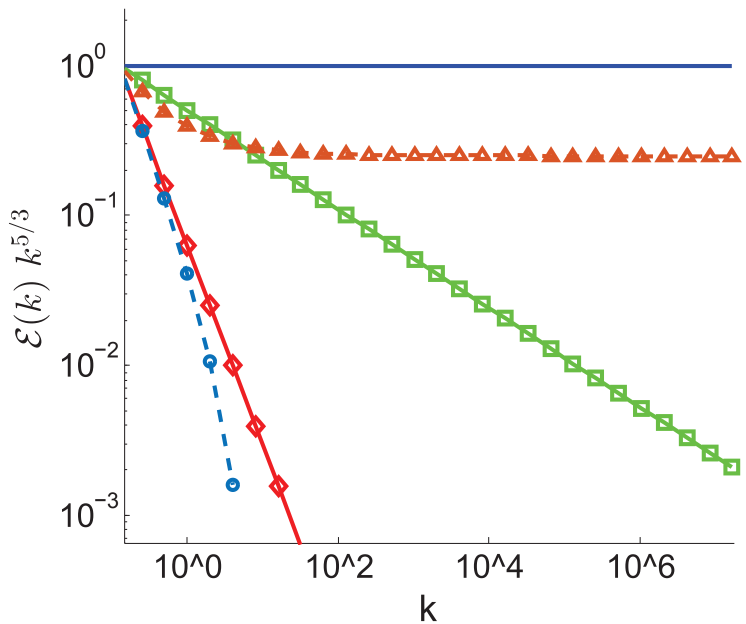
[4]D. Khomenko, V. S. Lvov, A. Pomyalov, and I. Procaccia, Counterflow decoupling in superfluid turbulence. arXiv:1509.03566 (Sept. 2015)

$$\frac{\partial \mathcal{E}_{\rm s}(k,t)}{2\,\partial t} + \mathcal{NL}_{\rm s} \;=\; \alpha\kappa\mathcal{L}\big[\mathcal{E}_{\rm ns}(k,t)-\mathcal{E}_{\rm s}(k,t)\big] \simeq -\alpha\kappa\mathcal{L}\mathcal{E}_{\rm s}(k,t)\big[1-D(k)\big]\,, \tag{8a}$$

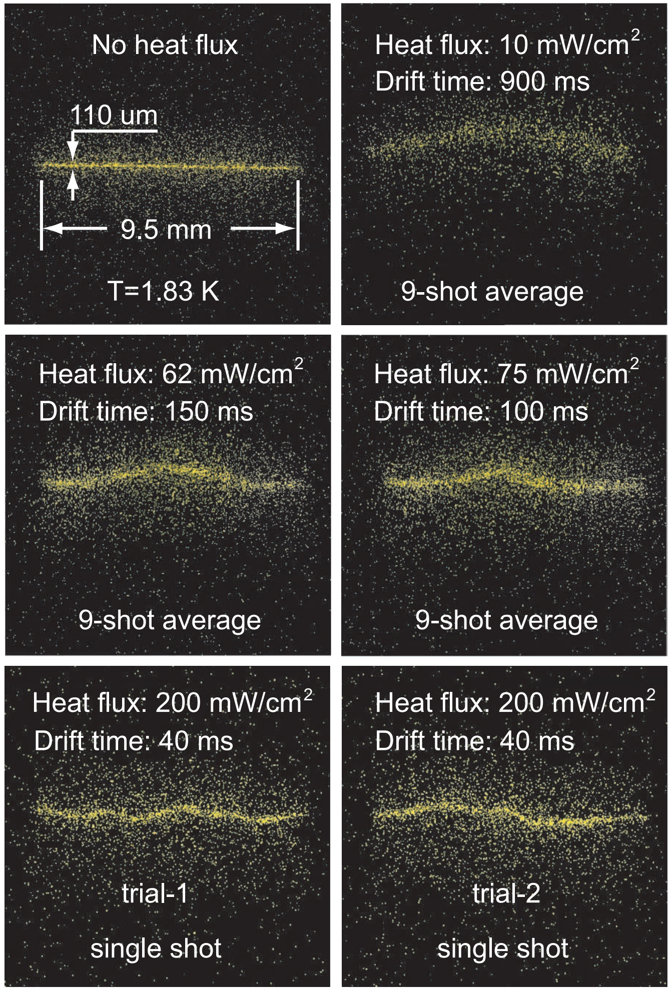
$$\frac{\partial \mathcal{E}_{\rm n}(k,t)}{2\,\partial t} + \mathcal{NL}_{\rm n} \;=\; \frac{\alpha\kappa\mathcal{L}\rho}{\rho_{\rm s}}\big[\mathcal{E}_{\rm ns}(k,t)-\mathcal{E}_{\rm n}(k,t)\big] \simeq -\frac{\alpha\kappa\mathcal{L}\rho}{\rho_{\rm s}}\big[\mathcal{E}_{\rm n}(k,t)\big[1-D(k)\big]\,. \tag{8b}$$

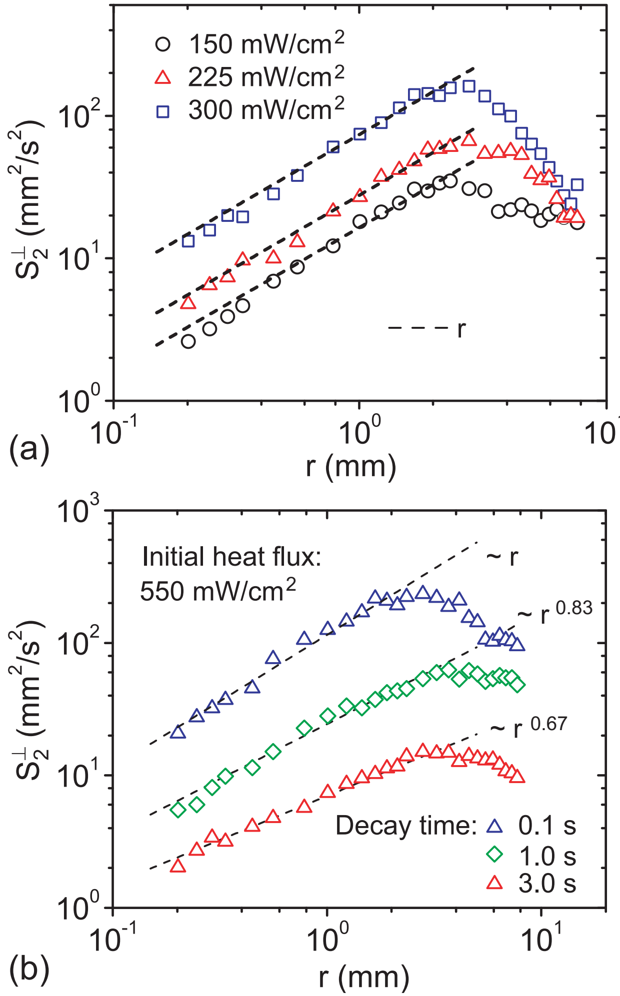
Here $\mathcal{NL}_{\rm s,n}$ are nonlinear terms which we do not specified at this stage of the research. For $k \gg k_\times$, $D(k) \ll 1$ and situation become similar to the equation for $\mathcal{E}_{\rm s}$ for the superfluid turbulence in ^{3}He, where mutual friction drastically suppresses the energy spectrum $\mathcal{E}_{\rm s}(k)$. Instead of classical Kolmogorov spectrum $\mathcal{E}(k) \propto k^{-5/3}$ Lvov, Nazarenko and Volovik (LNV) (JETP Letters, 2004) found the spectra

$$\boldsymbol{\mathcal{E}_{\rm s}(k) \propto \frac{1}{k^{5/3}\left[\frac{1}{k^{2/3}} \pm \frac{1}{k_{\rm cr}^{2/3}}\right]^2}}\,, \qquad \text{critical: } \propto k^{-3},\ k_{\rm cr}\to\infty, \text{ subcrititical (with } -\text{), supercritical } (+).$$



Wei Guo (Florida) visualization experiment in counter-flow ^{4}He turbulence [5]



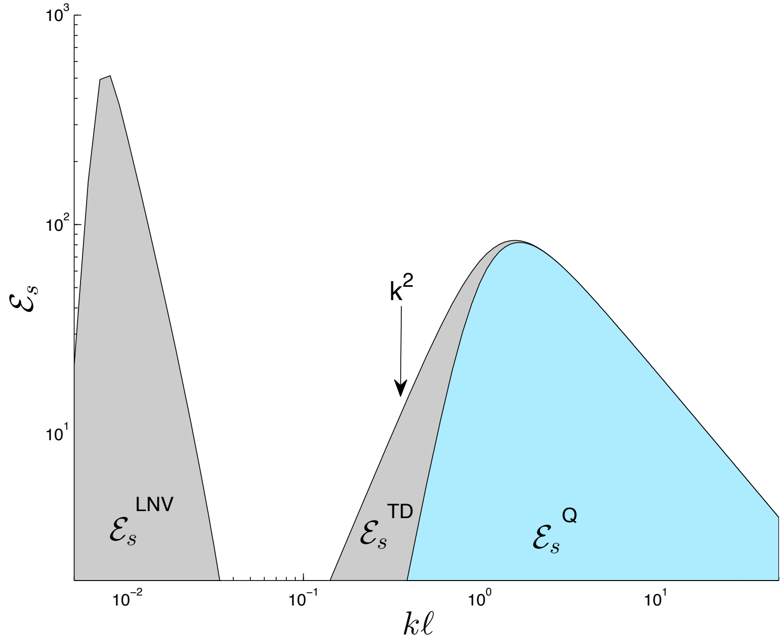


results in transversal 2-nd order normal-fluid structure function $S_2(r) \propto r$, corresponding to $E(k) \propto k^{-2}$!!. There are no reasonable explanation for this simple result! (if it is true?)

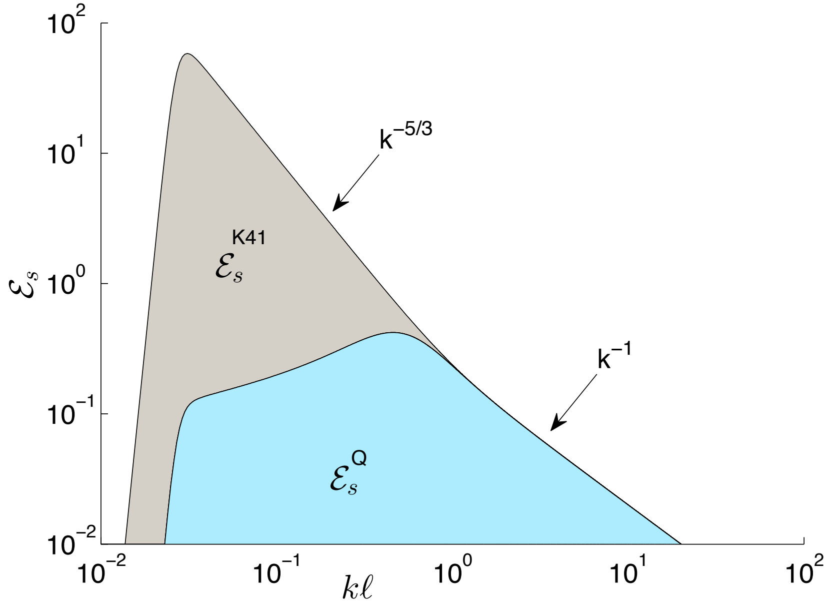
[5] A. Marakov, J. Gao, W. Guo, S. W. Van Sciver, G. G. Ihas, D. N. McKinsey, and W. F. Vinen, Visualization of the normal-fluid turbulence in counterflowing superfluid ^{4}He, Phys. Rev. B 91, 094503 (2015).

Sketch of the superfluid turbulent energy spectra

Counter- & super-flow stationary spectra



Late time asymptotics

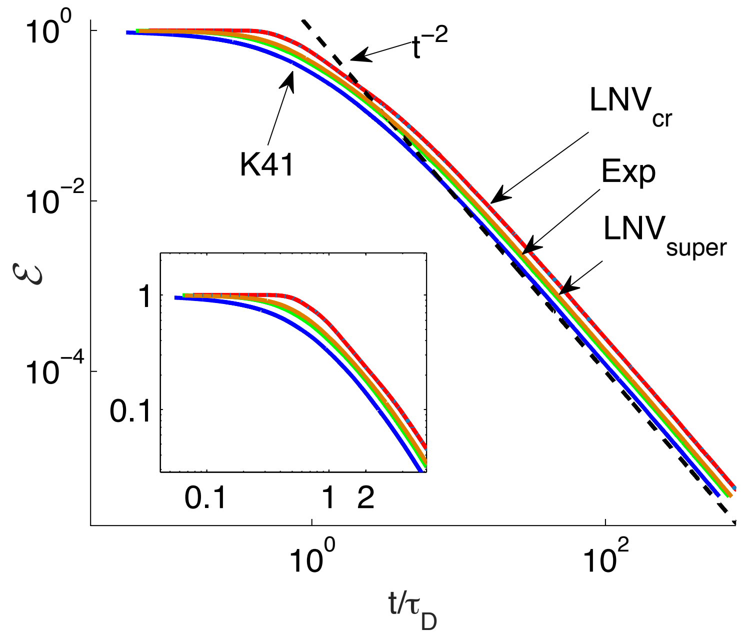


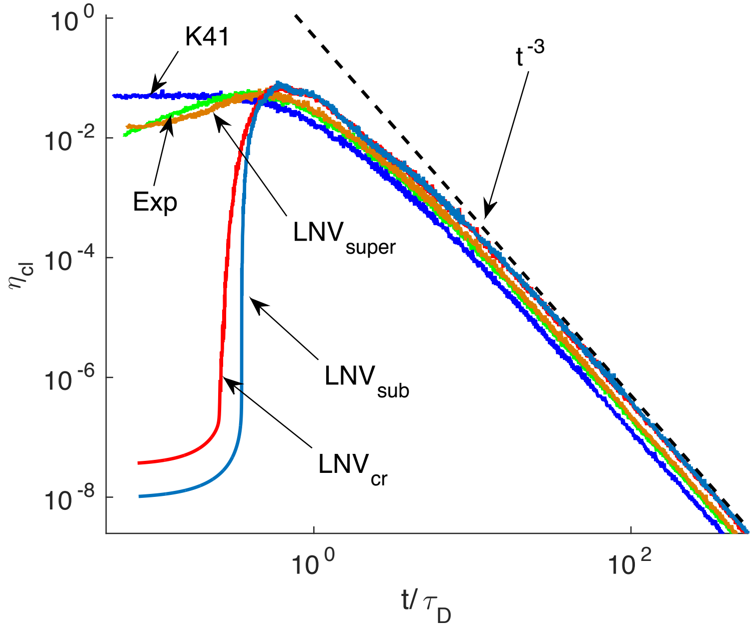
After switching off the counter- or super-flow the stationary energy spectrum of super-/counter-flow (left panel) evolve to the spectrum, shown in right panel, switching on the energy flux toward quantum vortex tangle after some delay.

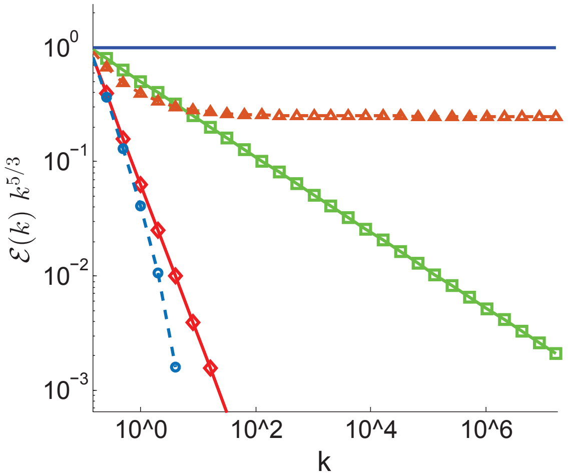
Numerical simulations of the classical turbulence decay by Sabra-shell model

After ensemble averaging over 10^4 realizations we got time dependence of

Left: total energy (with t^{-2} asymptotics) Right: Energy flux toward large k (with t^{-3} asymptotics)







For K41 initial condition (IC) – solid blue lines there are no delay in the energy dissipation. For LNV critical (red line) and sub-critical (blue dashed line) there is clear delay with sharp switching on, while for weakly localized IC ("experimental" k^{-2} spectrum (green) and LNV super-critical IC orange dashed lines) there is smooth switching on.

Perspectives and Summary

These days it is very timing and principally important to perform and analyze DNS of two-fluid, gradually damped (by the effective superfluid viscosity ν'_s) HVBK equations

$$\left[\frac{\partial}{\partial t} + (\boldsymbol{U}_s + \boldsymbol{u}_s)\cdot\boldsymbol{\nabla} - \nu'_s\,\Delta\right]\boldsymbol{u}_s - \boldsymbol{\nabla} p_s = \alpha\kappa(\boldsymbol{u}_n - \boldsymbol{u}_s)\mathcal{L}\,,$$
$$\left[\frac{\partial}{\partial t} + (\boldsymbol{U}_n + \boldsymbol{u}_n)\cdot\boldsymbol{\nabla} - \nu_n\,\Delta\right]\boldsymbol{u}_s - \boldsymbol{\nabla} p_n = \alpha\frac{\rho_s}{\rho_n}\kappa(\boldsymbol{u}_s - \boldsymbol{u}_n)\mathcal{L}\,,$$

for co- and counter-flow homogeneous superfluid turbulence in a periodic box with measuring the stationary energy spectra $E_{ss}(k) = 4\pi k^2\left\langle|\boldsymbol{u}_s(\boldsymbol{k})|^2\right\rangle$ and $E_{nn}(\boldsymbol{k}) = 4\pi k^2\left\langle|\boldsymbol{u}_n(\boldsymbol{k})|^2\right\rangle$ together with $\times$-velocity correlations $E_{ns}(k) = 4\pi k^2\left\langle\boldsymbol{u}_n(\boldsymbol{k})\cdot\boldsymbol{u}_s(\boldsymbol{k})\right\rangle$:

- In co-flow ($U_n = U_s = 0$) this will clarify the issue of intermittency, velocity coupling and energy exchange between the normal- and super-fluid subsystems;
- In counter-flow ($\boldsymbol{U}_{ns} = \boldsymbol{U}_n - \boldsymbol{U}_s \neq 0$) this will allows to find new energy spectra of the counter-flowing super-fluid turbulence and to clarify the counter-flow decoupling of the normal- and super-fluid turbulent velocity fluctuations.

Much more experimental, analytical and numerical studies are required to achieve desired level of understanding of superfluid turbulence